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N=1, 4D Supermembrane from 11D.

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Abstract: The action of the 11D supermembrane with nontrivial central charges compactified on a 7D toroidal manifold is obtained. It describes a supermembrane evolving in a 4d Minkowski space-time. The action is invariant under additional symmetries in comparison to the supermembrane on a Minkowski target space. The hamiltonian in the LCG is invariant under conformal transformations on the Riemann surface base manifold. The spectrum of the regularized hamiltonian is discrete with finite multiplicity. Its resolvent is compact. Susy is spontaneously broken, due to the topological central charge condition, to four supersymmetries in 4D, the vacuum belongs to an N=1 supermultiplet. When assuming the target-space to be an isotropic 7-tori, the potential does not contain any flat direction, it is stable on the moduli space of parameters.

Keywords: supermembrane, 4D, moduli, supersymmetry.

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1. Introduction

The non-perturbative quantization of String Theory is still an open problem which has received recently a lot of attention. It can be reformulated in terms of the quantization of the M-theory in 11 dimensions. In spite of the advances towards its quantization ([1]-[10]), M-theory is a not well understood theory. The goal to achieve is to obtain a consistent quantum theory in 4D with an $N = 1$ or $N = 0$ supersymmetries, moduli free, in agreement with the observed 4D physics. Attempts to formulate the theory in 4D have been done in the supergravity approach [11, 12], including fluxes [13, 14], but so far no exact formulation has been found. The theory when compactified to a 4D model contains many vacua due to the presence of moduli fields. Stabilizacion of these moduli is an important issue to be achieved. For some interesting proposals see [13, 15]. In the following we will consider the $11D$ M2-brane with irreducible wrapping on the compact sector of the target manifold [16]. This implies the existence of a non trivial central charge in the supersymmetric algebra. Its spectral properties have been obtained in several papers [5]-[10]. Summarizing them: classically it does not contain singular configuration with zero energy, and quantically its regularized bosonic and supersymmetric hamiltonians have a discrete

spectrum with finite multiplicity. This proof has been extended to the exact case for the bosonic sector of a supermembrane with central charges compactified in a 2-torus. Extensions of this proof to the supersymmetric case and other target manifolds are currently under study.

The purpose of this paper is to construct the action for the supermembrane with nontrivial central charges compactified on a T^7 and analyze its physical properties. The action describes a supermembrane evolving in a 4D Minkowski space. It is invariant under SUSY with a Majorana 32 component spinor parameter however the symmetry is spontaneously broken to a $N = 1$ theory in 4 dimensions when the minimal configuration is fixed. A detailed analysis of the compactification properties will be given elsewhere [17]. When the compactification manifold is considered to be an isotropic T^7 i.e. all the radii are equal, the potential has no flat directions, it is stable in the moduli space of parameters.

The paper is structured as follows: section 2 is devoted to explain the construction of the supermembrane with central charges. In section 3 we show how the compactification on the remaining 7-coordinate is performed to obtain a $4D$ action of the supermembrane. Section 4 shows explicitly the spontaneous breaking of supersymmetry. In section 5 we perform the analysis of the spectral properties of the theory. In section 6 we analyze some physical properties as mass generation and moduli stabilization due to the topological condition and finally we present our conclusions in section 7.

2. D=11 Supermembrane with central charges on a $M_5 \times T^6$ target manifold

The hamiltonian of the $D = 11$ Supermembrane [18] may be defined in terms of maps X^M, $M = 0, \dots, 10$, from a base manifold $R \times \Sigma$, where Σ is a Riemann surface of genus g onto a target manifold which we will assume to be $11-l$ Minkowski $\times$ l-dim Torus. The canonical reduced hamiltonian to the light-cone gauge has the expression

$$\int_\Sigma \sqrt{W} \left(\frac{1}{2} \left(\frac{P_M}{\sqrt{W}} \right)^2 + \frac{1}{4} \{X^M, X^N\}^2 + \text{ Fermionic terms} \right) \tag{2.1}$$

subject to the constraints

$$\phi_1 := d(\frac{p_M}{\sqrt{W}} dX^M) = 0 \tag{2.2}$$

and

$$\phi_2 := \oint_{C_s} \frac{P_M}{\sqrt{W}} \quad dX^M = 0, \tag{2.3}$$

where the range of M is now $M = 1, \dots, 9$ corresponding to the transverse coordinates in the light-cone gauge, C_s, $s = 1, \dots, 2g$ is a basis of 1-dimensional homology on Σ,

$$\{X^M, X^N\} = \frac{\epsilon^{ab}}{\sqrt{W(\sigma)}} \partial_a X^M \partial_b X^N. \tag{2.4}$$

$a, b = 1, 2$ and σ^a are local coordinates over Σ. $W(\sigma)$ is a scalar density introduced in the light-cone gauge fixing procedure. ϕ_1 and ϕ_2 are generators of area preserving diffeomorphisms. That is

$$\sigma \to \sigma' \quad \to \quad W'(\sigma) = W(\sigma).$$

When the target manifold is simply connected dX^M are exact one-forms.

The $SU(N)$ regularized model obtained from (2.1) was shown to have conti nuous spectrum from $[0, \infty)$, [2],[3],[1].

This property of the theory relies on two basic facts: supersymmetry and the presence of classical singular configurations, string-like spikes, which may appear or disappear without changing the energy of the model but may change the topology of the world-volume. Under compactification of the target manifold generically the same basic properties are also present and consequently the spectrum should be also continuous [4]. In what follows we will impose a topological restriction on the configuration space. It characterizes a $D = 11$ supermembrane with non-trivial central charges generated by the wrapping on the compact sector of the target space [5],[6],[8],[10]. Following [9] we may extend the original construction on a $M_9 \times T^2$ to $M_7 \times T^4$, $M_5 \times T^6$ target manifolds by considering genus $1, 2, 3$ Riemann surfaces on the base respectively. Under such correspondence there exists a minimal holomorphic immersion from the base to the target manifold. The image of Σ under that map is a calibrated submanifold of T^2, T^4, T^6 respectively. The model in those cases present additional interesting symmetries beyond the original ones [10].

We are interested in reducing the theory to a 4 dimensional model, we will then assume a target manifold $M_4 \times T^6 \times S^1$. The configuration maps satisfy:

$$\oint_{c_s} dX^r = 2\pi S^r_s R^r \quad r, s = 1, \dots, 6. \tag{2.5}$$

$$\oint_{c_s} dX^7 = 2\pi L_s R \tag{2.6}$$

$$\oint_{c_s} dX^m = 0 \quad m = 8, 9 \tag{2.7}$$

where S^r_s, L_s are integers and $R^r, r = 1, \ldots, 6$ are the radius of $T^6 = S^1 \times \cdots \times S^1$ while R is the radius of the remaining S^1 on the target. This conditions ensure that we are mapping Σ onto a $\Pi_{i=1}^7 S_i^1$ sector of the target manifold.

We now impose the central charge condition

$$I^{rs} \equiv \int_\Sigma dX^r \wedge dX^s = (2\pi R^r R^s)\omega^{rs} \tag{2.8}$$

where ω^{rs} is a symplectic matrix on the T^6 sector of the target.

For simplicity we take ω^{rs} to be the canonical symplectic matrix

$$M = \begin{pmatrix} 0 & 1 & & & & \\ -1 & 0 & & & & \\ & & 0 & 1 & & \\ & & -1 & 0 & & \\ & & & & 0 & 1 \\ & & & & -1 & 0 \end{pmatrix}. \tag{2.9}$$

It corresponds to the orthogonal intersection of the three $g = 1$,i.e. three toroidal T^2 supermembranes, the time direction being the intersecting space. The topological condition (2.8) does not change the field equations of the hamiltonian (2.1). In fact, any variation of I^{rs} under a change δX^r, single valued over Σ, is identically zero. In addition to the field equations obtained from (2.1), the classical configurations must satisfy the condition (2.8). It is only a topological restriction on the original set of classical solutions of the field equations. In the quantum theory the space of physical configurations is also restricted by the condition (2.8). The geometrical interpretation of this condition has been discussed in previous work [7],[16]. We noticed that (2.8) only restricts the values of S^r_s, which are already integral numbers from (2.5).

We consider now the most general map satisfying condition (2.8). A closed one-forms dX^r may be decomposed into the harmonic plus exact parts:

$$dX^r = M^r_s d\widehat{X}^s + dA^r \tag{2.10}$$

where $d\widehat{X}^s$, $s = 1, \ldots, 2g$ is a basis of harmonic one-forms over Σ. We may normalize it by choosing a canonical basis of homology and imposing

$$\oint_{c_s} d\widehat{X}^r = \delta^r_s. \tag{2.11}$$

We have now considered a Riemann surface with a class of equivalent canonical basis. Condition (2.5) determines

$$M^r_s = 2\pi R^r L^r_s. \tag{2.12}$$

dA^r are exact one-forms. We now impose the condition (2.8) and obtain

$$S_t^r \omega^{tu} S_u^s = \omega^{rs}, \tag{2.13}$$

that is, $S \in Sp(2g, Z)$. This is the most general map satisfying (2.8).

The natural election for $\sqrt{W(\sigma)}$ in this geometrical setting is to consider it as the density obtained from the pull-back of the Khäler two-form on T^6. We then define

$$\sqrt{W(\sigma)} = \frac{1}{2}\partial_a \widehat{X}^r \partial_b \widehat{X}^s \omega_{rs}. \tag{2.14}$$

$\sqrt{W(\sigma)}$ is then invariant under the change

$$d\widehat{X}^r \to S_s^r d\widehat{X}^s, \quad S \in Sp(2g, Z) \tag{2.15}$$

But this is just the change on the canonical basis of harmonics one-forms when a biholomorphic map in Σ is performed changing the canonical basis of homology. That is, the biholomorphic (and hence diffeomorphic) map associated to the mo du lar transformation on a Teichmüller space. We thus conclude that the theory is invariant not only under the diffeomorphisms generated by ϕ_1 and ϕ_2 but also under the diffeomorphisms, biholomorphic maps, changing the canonical basis of homology by a modular transformation.

The theory of supermembranes with central charges in the light cone gauge (LCG) we have constructed depends then on the moduli space of compact Riemanian surfaces M_g only. It may be defined on the conformal equivalent classes of compact Riemann surfaces. It shares this property with String Theory, although the supermembrane theory is still restricted by the area preserving constraints, there are area preserving diffeomorphisms which are not conformal mappings. In addition, the supermembrane depends on the moduli identifying the holomorphic immersion from M_g to the target manifold. This is an interesting moduli space already considered in a different context in [19].

Having identified the modular invariance of the theory we may go back to the general expression of dX^r, we may always consider a canonical basis in away that

$$dX^r = R^r d\widehat{X^r} + dA^r. \tag{2.16}$$

the corresponding degrees of freedom are described exactly by the single-valued fields A^r. After replacing this expression in the hamiltonian (2.1) we obtain,

$$
\begin{aligned}
H = & \int_\Sigma \sqrt{W} d\sigma^1 \wedge d\sigma^2 [\frac{1}{2}(\frac{P_m}{\sqrt{W}})^2 + \frac{1}{2}(\frac{\Pi^r}{\sqrt{W}})^2 + \frac{1}{4}\{X^m, X^n\}^2 + \frac{1}{2}(\mathcal{D}_r X^m)^2 + \frac{1}{4}(\mathcal{F}_{rs})^2 \\
& + \int_\Sigma \mathcal{F} + (2\pi)^4 (R^r R^s)^4 \int_\Sigma \sqrt{W} (D_r \widehat{X})^2 + \Lambda(\mathcal{D}_r(\frac{\Pi_r}{\sqrt{W}}) + \{X^m, \frac{P_m}{\sqrt{W}}\})] \\
& + \int_\Sigma \sqrt{W}[-\overline{\Psi}\Gamma_- \Gamma_r \mathcal{D}_r \Psi] + \overline{\Psi}\Gamma_- \Gamma_m \{X^m, \Psi\} + \Lambda\{\overline{\Psi}\Gamma_-, \Psi\}]
\end{aligned}
\tag{2.17}
$$

where $\mathcal{D}_r X^m = D_r X^m + \{A_r, X^m\}$, $\mathcal{F}_{rs} = D_r A_s - D_s A_r + \{A_r, A_s\}$, $D_r = 2\pi R^r \frac{\epsilon^{ab}}{\sqrt{W}} \partial_a \widehat{X}^r \partial_b$ and P_m and Π_r are the conjugate momenta to X^m and A_r respectively. $\mathcal{D}_r$ and $\mathcal{F}_{rs}$ are the covariant derivative and curvature of a symplectic noncommutative theory [16],[6], constructed from the symplectic structure $\frac{\epsilon^{ab}}{\sqrt{W}}$ introduced by the central charge. We will take the integral of the curvature to be zero and the volume term corresponds to the value of the hamiltonian at its ground state. The last term represents its supersymmetric extension in terms of Majorana spinors. The physical degrees of the theory are the X^m, A_r, Ψ_α they are single valued fields on Σ.

In [9] a T^4 compactification sector was considered. Its hamiltonian was expressed in terms of a different frame for the compactified sector on the T^4 torus. In that case the pullback is performed directly with the harmonic modes $d\widehat{X}^r$ while in the present formulation the metric on that sector is δ_{rs} and the pullback should be performed with $G^{1/2} d\widehat{X}$, G is the constant matrix introduced in [9]. In both cases the same scalar density $\sqrt{W(\sigma)}$ is obtained.

3. Compactification on the remaining S^1

The analysis of the compactification on the remaining S^1 may be performed directly in the above formalism or by considering its dual formulation in term of $U(1)$ gauge fields. We will discuss both approaches.

In the first case, we may solve the condition (2.6), we obtain

$$
dX^7 = R L_s d\widehat{X}^s + d\widehat{\phi} \tag{3.1}
$$

where $d\widehat{\phi}$ is an exact 1-form and $d\widehat{X}^s$ as before are a basis of harmonic 1-forms over Σ. For the discretness analysis at is more convenient to express dX^7 in terms of the solution of the "covariant" laplacian over Σ:

$$
\mathcal{D}_r \mathcal{D}_r \widetilde{X} = 0 \tag{3.2}
$$

where $\mathcal{D}_r$, $r = 1, \ldots, 2g$ were defined in the previous section. There are $2g$ independent solutions of (3.2). In fact, $d\widetilde{X}$ is necessarily a linear combination of the

basis of harmonic 1-forms plus exact forms. For each $d\widehat{X}^s$ there exists a unique ϕ^s, single-valued over Σ such that

$$D_r D_r \widehat{X}^s + D_r D_r \phi^s = 0. \tag{3.3}$$

The most general solution for $\widetilde{X}_1$ satisfying $D_r D_r \widetilde{X}_1 = 0$ is then

$$d\widetilde{X}_1 = L_s (d\widehat{X}^s + d\phi^s), \tag{3.4}$$

since the only solution in terms of pure exact forms is the trivial one. We notice also that $d\widehat{X}^s + d\phi^s$ $s = 1, \dots, 2g$ are linearly independent. The most general solution for $\mathcal{D}_r \mathcal{D}_r \widetilde{X} = 0$, at least perturbatively in A_r is of the same form (3.4) since all new contributions to the solution are exact. We may rewrite (3.1) in the form

$$dX^7 = R L_s d\widetilde{X}^s + d\phi, \tag{3.5}$$

we notice that L_s are the same as in (2.12). The only change is in the exact 1-forms.

We may now analyze the contribution of the dX^7 field to the hamiltonian. In addition to its conjugate momentum, which appears quadratically we have

$$V_7 = \left\langle (\mathcal{D}_r X^7)^2 + \{X^m, X^7\}^2 \right\rangle = \left\langle (L_s \mathcal{D}_r \widetilde{X}^s)^2 + (\mathcal{D}_r \phi)^2 + \{X^m, X^7\}^2 \right\rangle \tag{3.6}$$

where we have explicitly used (3.2).

We then obtain the bound

$$V_7 \geq \left\langle (\mathcal{D}_r \phi)^2 + \{X^m, X^7\}^2 \right\rangle \geq (\mathcal{D}_r \phi)^2 \tag{3.7}$$

which directly shows that the winding corresponding to dX^7 does not affect the qualitative properties of the spectrum of (2.17). The first term in (3.6)

$$\left\langle L_s (\mathcal{D}_r \widetilde{X}^s)^2 \right\rangle \tag{3.8}$$

corresponds, for a given A_r, to the value of the original quadratic term $\langle (\mathcal{D}_r X^7)^2 \rangle$ in the action evaluated at a configuration that minimizes it. This is the case, since in the usual Dirichlet internal product

$$(A, B) = \int_\Sigma \mathcal{D}_r A \mathcal{D}_r B \tag{3.9}$$

defined on fields modulo constants, the exact and harmonic functions are orthogonal. Exactly the same decomposition as in (3.6) occurs when evaluating the sum over isomorphisms class of line bundles L in the partition function of an abelian

gauge field. The sum over L gives a generalized θ function [19, 20]. The third term in (23)contributes with L_s^2 to the coefficient of the quadratic mass terms $(D_s X^m)^2$ the total factor being $(1 + L_s^2)$. The first term $\left\langle (\mathcal{D}_r \widetilde{X}^s)^2 \right\rangle$ contains a complicated dependence on A_r, if one is interested in analyzing it is better to start with the decomposition (3.1) instead of (3.5). For the purpose of our analysis we may neglect this complicated positive term and consider the hamiltonian with a potential contribution given by the right hand side of (3.7). The inequality (3.7) will ensure that the qualitative properties of the latest are also valid for the original complete hamiltonian.

We will now construct the dual formulation to (2.17) when dX^7 is restricted by the condition (2.6) ensuring that X^7 takes values on S^1. We follow [20], given

$$\mathcal{L} = p_I \dot{X}^I + p\dot{X} - \mathcal{H}(p_I, X^I, p, x) \tag{3.10}$$

where

$$\oint_{C_s} dX = RL_s \tag{3.11}$$

and the dependence on X is only through its derivatives $\partial_\lambda X$ and construct

$$\left\langle \widehat{\mathcal{L}} + W_\lambda F_{\mu\nu} \epsilon^{\lambda\mu\nu} \right\rangle \tag{3.12}$$

where

$$\widehat{\mathcal{L}} = p_I \dot{X}^I + pW_0 - \mathcal{H}(p_I, X^I, p, W_a) \tag{3.13}$$

the elimination of W_0, through its field equation or directly from a gaussian integral in the functional integration, yields the dual action

$$\widetilde{\mathcal{L}} = p_I \dot{X}^I + \Pi^a \dot{A}_a + A_0 \partial_a \pi^a - \mathcal{H}(p_I, X^I, F_{ab}\epsilon^{ab}, -\frac{1}{2}\epsilon_{ba}\Pi^b). \tag{3.14}$$

It is already in a canonical hamiltonian formulation. The new hamiltonian is obtained from the original one by making the above replacements. Notice that there is no assumption on the structure of $\mathcal{H}$, it is not necessarily quadratic. In our particular case the dependence on p and W_a is quadratic. Condition (3.11) becomes now

$$\oint_{C_s} (-\frac{1}{2}\epsilon_{ba}\Pi^b) d\sigma^a = \frac{1}{R} m_s \tag{3.15}$$

where m_s are integers.

We notice that A_s is not a connection in a line bundle over Σ. In fact the condition

$$\int_\Sigma F_{ab} d\sigma^a \wedge d\sigma^b = 2\pi n \tag{3.16}$$

is not necessarily satisfied. In order to have a connection on line bundle over Σ one should require a periodic euclidean time on the functional integral formulation. In that case the condition (2.6), where now the basis of one-dimensional homology includes the additional S^1, ensures that $F_{\mu\nu}$ is the curvature of a one-form connection over the three dimensional base manifold. Under this assumption the condition (2.6) for any L_s implies summation over all $U(1)$ principle bundles. The contribution of this summation of the partition function is a generalized θ function [19] arising from the evaluation of the abelian action at minimizing configurations, that is monopole-type solutions [21].

The final expression of the dual formulation to (2.17) when X^7 is wrapped on a S^1, condition (2.6), is

$$\begin{aligned}
H_d = &\int_\Sigma \sqrt{w} d\sigma^1 \wedge d\sigma^2 [\frac{1}{2}(\frac{P_m}{\sqrt{W}})^2 + \frac{1}{2}(\frac{\Pi^r}{\sqrt{W}})^2 + \frac{1}{4}\{X^m, X^n\}^2 + \frac{1}{2}(\mathcal{D}_r X^m)^2 \\
&+ \frac{1}{4}(\mathcal{F}_{rs})^2 + \frac{1}{2}(F_{ab}\frac{\epsilon^{ab}}{\sqrt{W}})^2 + \frac{1}{8}(\frac{\Pi^c}{\sqrt{W}}\partial_c X^m)^2 + \frac{1}{8}[\Pi^c \partial_c(\widehat{X}_r + A_r)]^2] \\
&+ \Lambda(\{\frac{P_m}{\sqrt{W}}, X^m\} - \mathcal{D}_r(\frac{\Pi^r}{\sqrt{W}}) - \frac{\Pi^c}{2\sqrt{W}}\partial_c(F_{ab}\frac{\epsilon^{ab}}{\sqrt{W}})) + \lambda \partial_c \Pi^c] \\
&+ \int_\Sigma \sqrt{W}[-\overline{\Psi}\Gamma_- \Gamma_r \mathcal{D}_r \Psi + \overline{\Gamma}_- \Gamma_m \{X^m, \Psi\} + 1/2\overline{\Psi}\Gamma_7 \Pi^b \partial_b \Psi] + \Lambda\{\overline{\Psi}\Gamma_-, \Psi\}
\end{aligned}$$

The term

$$\begin{aligned}
&\frac{1}{2}(\frac{P_m}{\sqrt{W}})^2 + \frac{1}{4}\{X^m, X^n\}^2 + \frac{1}{2}(F_{ab}\frac{\epsilon^{ab}}{\sqrt{W}})^2 + \frac{1}{8}(\frac{\Pi^c}{\sqrt{W}}\partial_c X^m)^2 \\
&+ \Lambda(\{\frac{P_m}{\sqrt{W}}, X^m\} - \frac{1}{2}\Pi^c \partial_c(F_{ab}\frac{\epsilon^{ab}}{\sqrt{W}})) + \lambda \partial_c \Pi^c
\end{aligned}$$

describe the canonical density of a Dirac-Born Infeld theory in terms of $G_{ab} = \partial_a X^m \partial_b X^m$ and F_{ab}. In the full theory with the hamiltonian H_d, there are additional interacting terms describing the coupling to the sector wrapped onto a T^6. We considered the M2 with all physical configurations wrapped in an irreducible way on a $T^6 \times S^1$ target space. If we now take the compactified sector of the target to be $T^7 = (S^1)^7$,we should then consider all possible decompositions of the form $T^6 \times S^1$.The Hilbert space of physical configurations is then enlarged by considering all possible holomorphic immersions and their corresponding physical states in terms of single valued fields over the base manifold, as explained in section 2. The breaking of susy induced by the ground state follows in the same way.

4. $N = 1$ supersymmetry

The topological condition associated to the central charge determines an holomorphic minimal immersion from the g-Riemann surface to the 2g-torus target manifold. This minimal immersion is directly related to the BPS state that minimizes the hamiltonian. When we start with the $g = 1$ and T^2 on the target space the ground state preserves $\frac{1}{2}$ of the original supersymmetry with parameter a 32-component Majorana spinor. When we consider our construction for a $g = 2, 3$ and T^4, T^6 torus on the target, the analysis of the SUSY preservation becomes
exactly the same as when considering orthogonal intersection of 2-branes with the time direction as the intersecting direction [22]. The SUSY of the ground state preserves $\frac{1}{4}$, $\frac{1}{8}$ of the original SUSY. The ground state in all these cases corresponds to

$$\Psi = 0 \quad X^m = 0 \quad X_i^r = \widehat{X}_i^r \tag{4.1}$$

The preservation of the ground state implies the breaking of the supersymmetry. In the light cone gauge, we end up when $g = 3$ with $\frac{1}{8}$ of the original SUSY, that is one complex grassmann parameter corresponding to a $N = 1$ light-cone SUSY multiplet.

The action is invariant under the whole light-cone SUSY. There is a whole class of minima for the hamiltonian, corresponding to

$$\begin{aligned}
&\Psi = \epsilon_1 + \epsilon_2 \\
&X^r = \widehat{X}^r + i\bar{\epsilon}_2\Gamma\epsilon_1 \\
&X^m = i\bar{\epsilon}_2\Gamma^m\epsilon_1.
\end{aligned}$$

However when the vacuum is spontaneously fixed to one of them, the SUSY is broken at the quantum level up to $N = 1$ when the target is $M_5 \times T^6$. There is no further breaking when we compactify the additional S^1, to have a target $M_4 \times T^6 \times S^1$.

5. Discreteness of the spectrum

We consider a gauge fixing procedure on a BFV formulation of the theory. Several gauge conditions are appropriate to analyze the qualitative properties of the spectrum. We may impose as in ([6],[8]) a gauge choice once a normal basis of functions over Σ is introduced in the theory. We may otherwise consider a Coulomb gauge condition $D_r A_r = 0$. We may solve it in terms of longitudinal and transverse modes in the usual way, together with a resolution of the first class constraint, the Gauss constraint. In this case once all the canonical hamiltonian is expressed in terms of the transverse canonical modes one is left with a positive but complicated term arising from the square of the momenta terms after the decoupling of the longitudinal term has been obtained. It is of the form

$$D_r \Pi^L D_r \Pi^L \tag{5.1}$$

where Π^L is the longitudinal part of the momenta Π_r, and Π^L has to be replaced by the solution of the constraint. In what follows we may eliminate such positive term, since the discreteness of the lower bound operator ensures the same property for the original hamiltonian. The same argument was used in [6].

We may also consider a gauge fixing condition

$$\chi \equiv a D_r A_r + B \tag{5.2}$$

where B is the BRST transformed of the antighost field while a is a real number which can be chosen in a way to cancel the $(D_r A_r)^2$ contribution from the $\mathcal{F}^2$ term in the hamiltonian. After a redefinition of B it decouples from the functional integral, we end up with a canonical formulation in terms of the square of all the momenta together with the quadratic mass terms for each mode in the formulation. An important aspect to mention is that in all these cases the ghost fields do not decouple from the action, however the contributions are always linear on the configuration variables. Theorem 2 in [23] ensure that this ghost contribution does not change the discreteness properties of the canonical formulation. The discussion of the spectral properties of the hamiltonian is then largely simplified by those consideration. We may reduce to the physical degrees of freedom or we may enlarge the phase space as in BFV canonical formalism, in both cases the analysis reduces to a Schröendiger operator with quadratic mass terms and positive potential. We may consider for example,using (3.7)

$$\begin{aligned}\widehat{H} = \int_\Sigma \sqrt{W}[\frac{1}{2}(\frac{P_m}{\sqrt{W}})^2 + \frac{1}{2}(\frac{P}{\sqrt{W}})^2 + \frac{1}{2}(\frac{\Pi^r}{\sqrt{W}})^2 + \frac{1}{4}\{X^m, X^n\}^2 \\ + \frac{1}{4}\{X^m, X^7\}^2 + \frac{1}{2}(\mathcal{D}_r X^m)^2 + \frac{1}{2}(\mathcal{D}_r \phi)^2 + \frac{1}{4}(\mathcal{F}_{rs})^2 + \frac{1}{2}(\mathcal{D}_r A_r)^2]\end{aligned}$$

where P is the conjugate momenta of ϕ, the contribution of the compactification on S^1. This hamiltonian is exactly of the form considered in [6],[8][23]. We may then apply , for a regularized version of it, the results developed there.

We consider, in the usual way, a decomposition of all scalar fields over Σ in terms of an orthonormal discrete basis $Y_A(\sigma^1, \sigma^2)$. It is relevant in this approach to consider a compact closed Riemann surface Σ

$$\begin{aligned}X^m(\Sigma, \sigma, \tau) =& X^{mA}(\tau) Y_A(\sigma^1, \sigma^2) \\ A_r(\Sigma, \sigma, \tau) =& A_r^A(\tau) Y_A(\sigma^1, \sigma^2) \\ \phi(\Sigma, \sigma, \tau) =& \phi^A(\tau) Y_A(\sigma^1, \sigma^2)\end{aligned}$$

The symplectic bracket is also a scalar over Σ, hence it must be rewritten in terms of the basis

$$\{Y_A, Y_B\} = f_{AB}^{C} Y_C \tag{5.3}$$

after defining

$$\int_{\Sigma} Y_A Y_B = \eta_{AB} \tag{5.4}$$

we get

$$\int_{\Sigma} \{Y_A, Y_B\} Y_C = f_{ABC}, \tag{5.5}$$

f_{ABC} are consequently completely antisymmetric. f_{AB}^{C} are the structure constant of the area preserving diffeomorphism. We then replace those expressions into the hamiltonian density and integrate the σ^1, σ^2 dependence. We obtain then a formulation of the operator in terms of the τ dependent modes only. We now consider a truncation of the operator, that is we restrict the range of the indices A, B, C to a finite set N and introduce constants $f_{AB}^{N\ \ C}$ such that

$$lim_{N\to\infty} f_{AB}^{N\ \ C} = f_{AB}^{C} \tag{5.6}$$

In [2],[3], $f_{AB}^{N\ \ C}$ are the structure constants of $SU(N)$, that is the truncated theory has also a gauge symmetry. In [8] for the supermembrane with central charges compactified on a T^2 the truncated theory in terms of $SU(N)$ structure constants also has a gauge symmetry. The algebra of first class constraints in both cases is the same. However, in the proof of the discretness of the spectrum in [8] the algebraic properties of $f_{AB}^{N\ \ C}$ do not play any role at all. We then proceed to the analysis of the spectrum of the truncated Schröedinger operator associated to $\widehat{H}$ without further requirements on the constants f^N:
i) The potential of the Schröedinger operator only vanishes at the origin of the configuration space:

$$V = 0 \to ||(X, A, \phi)|| = 0 \tag{5.7}$$

where $||.||$ denotes the euclidean norm in R^L. We notice that the original hamiltonian as well as $\widehat{H}$ are defined on fields up to constants.

ii) There exists a constant $M > 0$ such that

$$V(X, A, \phi) \geq M||(X, A, \phi))||^2 \tag{5.8}$$

Again, this bound arises from very general considerations. In fact, writing (X, A, ϕ) in polar coordinates

$$X = Rx \quad A = Ra \quad \phi = R\varphi \tag{5.9}$$

where $\theta \equiv (x, a, \varphi)$ is defined on the unit sphere, $||(x, a, \varphi)|| = 1$, we obtain

$$V(X, A, \phi) = R^2 P_\theta(R) \tag{5.10}$$

where

$$P_\theta(R) = R^2 k_1(\theta) + R k_2(\theta) + k_3(\theta) > 0 \tag{5.11}$$

with $k_3(\theta) > 0$, $k_1(\theta) \geq 0$ and $k_1(\theta) = 0 \Rightarrow k_2(\theta) = 0$. We then define

$$\mu(\theta) = min_R P_\theta(R), \tag{5.12}$$

it is continuous in θ and $\mu(\theta) > 0$. Using the compactness of the unit sphere we obtain

$$V(X, A, \phi) = R^2 P_\theta(R^2) \geq R^2 min_\theta = MR^2 \tag{5.13}$$

The Schröedinger operator is then bounded by an harmonic oscillator. Consequently it has a compact resolvent. We now use theorem 2 [23] to show that

i) The ghost and antighost contributions to the effective action,

ii) the fermionic contribution to the susy hamiltonian,

do not change the qualitative properties of the spectrum of the hamiltonian. In fact, both contributions are linear on the configuration variables.

In addition the susy contribution cancels the zero point energy of the bosonic oscillators even in the exact theory [24], [10].

We have shown then that the regularized compactified on the target space $M_4 \times T^6 \times S^1$ has a compact resolvent and hence a discrete spectrum with finite multiplicity. We expect the same result to be valid for the exact theory.

6. Physical properties

So far we have seeing that the action of the $N = 1$ supermembrane in four dimensions has a regularized discrete spectrum.

One of the characteristics of the theory is that due to the topological condition the fields acquire mass. This facts represents an alternative to Higgs mechanism since no Higgs particle is involved. There has been several mass generating mechanisms [25]-[28] in the literature. Since ours does not correspond strictly speaking to non of them although there are some resemblances, we will explain briefly just for

the sake of clarity. In here, the fields of the theory X^m, A_r, ϕ acquire mass via the vector fields $\widehat{X}_r$ defined on the supermembrane. Since those fields do not live in the target-space there is no violation of Lorentz invariance. In fact in resemblance with Scherk-Schwarz mechanism they induce a monodromy on the fields. It is important to point out that the number of degrees of freedom in 11D and in 4D is preserved, but just redistributed, contrary to the case of an effective compactification a la KK, where a tower of fields appear. This fact has the a advantage for many phenomenological purposes of maintaining the number of fields small.

One possible extra question is the analysis of moduli stabilization. Moduli are massless scalar fields that may parametrize the compactified geometry as well as different matter sectors. It can be distinguished two types of moduli: quantum moduli and classical moduli.

Quantum moduli of the theory, generically is not known although there has been some approximations for particular set-ups in which the different bunch of these moduli show the interpolation between different vacua with different gauge groups [29]. As we have pointed out along the text, the theory of quantum supermembranes with central charges we have constructed depends then on the moduli space of compact Riemanian surfaces M_g only. It is defined on the conformal equivalent classes of compact Riemann surfaces, and also depends on the moduli identifying the holomorphic immersion from M_g to the target manifold.

Generically the analysis of moduli fields have been performed at classical level in a supergravity approach [13]-[15]. It has being performed in effective 4D potentials of the supergravity approximations of M-inspired actions. The Kähler potential is expressed in terms of them [12],[14]. Since our approach is exact these terms do not appear, however the action posseses scalars that may lead to flat directions in the potential. We can distinguish between two types of scalars fields, those associated to the position of the supermembrane in the transverse dimensions, - analogous to what in String theory represent the open string moduli- and the scalars whose vevs parametrize the compact manifold - analogous to what in String theory represent the closed string moduli-.

We are going to analyze separately the two types of classical moduli. This decoupling approach is only justified iff the scales of stabilization (the masses of the moduli) are clearly different, otherwise the minimization with respect to the whole set of moduli (geometrical and of matter origin) should be performed. An exhaustive analysis of this fact is beyond the scope of this article. However some considerations still can be made:

At classical level the behaviour of the theory is known. The theory does not contain any string-like configuration. Let assume for the moment a compact manifold whose radii $R_1, \dots, R_7$ are fixed. The X^m that parametrize the position in the transverse dimensions of the supermembrane acquire mass due to the central charge condition so there are no flat directions in the scalar piece of the potential. The 7-component component has an induced effect due to the central charge condition through the quadratic coupling with the simplectic gauge fields A_r and gain also an effective mass. All of these type of moduli becomes stable.

So far we have considered the 7-torus to be rigid, in such a way that the $R_1 \dots, R_7$ are kept fixed and they do not appear in the metric (they would be the putative closed string moduli). If now we relax this condition and let them vary smoothly, we can ask ourselves if in principle it is possible to obtain a minimum. An heuristical argument to support moduli stabilization is the following: We are dealing in our construction with non trivial gauge bundles that can be represented as worldvolume fluxes [30]. Since for construction the mapping represent a minimal immersion on the target space they induce a similar effect that the one induced by the generalized calibration. Minimal calibrations take also into account the dependence on the base manifold, the Rieman surface chosen Σ. The condition of the generalized calibration -which shows the deformation of the cycles that are wrapped by the supermembrane- represent a condition for minimizing the energy [31]. It happens the same with the minimal inmersions. For a given induced flux,one may expect the volume to be fixed. The supermembrane with central charges is wrapping all of the 7-torus with the maximal amount of monopoles induced on it, so the overall geometric moduli in principle will be also stabilized.

Let us illustrate it, with the particular case of an isotropic torus, i.e. $R_1 = \cdots = R_7 = R_0$. We then obtain for the potential in $\widehat{H}$, the following expression

$$V = A + BR_0 + CR_0^2 + DR_0^4 \tag{6.1}$$

where $A \geq 0$, $C \geq 0$ and $D > 0$, with the following expressions

$$\begin{aligned}
D &= \frac{1}{4}\int_\Sigma \sqrt{W}\{\widehat{X}^r, \widehat{X}^s\}^2 \\
C &= \frac{1}{2}\int_\Sigma \sqrt{W}[(D_r X^m)^2 + (D_r\phi)^2 + (D_r A_s)^2 + \{X^m, \widehat{X}^s L_s\}^2] \\
B &= \int_\Sigma \sqrt{W}[\frac{1}{2}\{X^m, L_s\widetilde{X}^s\}\{X^m, \phi\} + D_r X^m\{A_r, X^m\} + D_r\phi\{A_r, \phi\} + \frac{1}{2}(D_r A_s - D_s A_r)\{A_r, A_s\}] \\
A &= \int_\Sigma \sqrt{W}[\frac{1}{4}\{X^m, X^n\}^2 + \frac{1}{4}\{X^m, \phi\}^2 + \frac{1}{2}\{A_r, X^m\}^2 + \frac{1}{2}\{A_r, \phi\}^2 + \{A_r, A_s\}^2]
\end{aligned}$$

where we have extracted the R_0 factor from the expression of the derivative D_r.

The quadratic mass terms contribute to the expression of C. C is zero if and only if (X^m, ϕ, A_s) are constants, in the equivalence class of zero. We obtain

$$\frac{d^2V}{dR_0^2} = 2C + 12DR_0^2 > 0 \tag{6.2}$$

consequently the problem is always stable with respect to the variations of R. We may have two possible minima: a minimum centered at $R_0 = 0$, or a minimum for $R_0 \neq 0$. The potential is globally stable with respect to the modulus R_0.

In the cases in which the radii are all different a more exhaustive analysis is need. We will consider it elsewhere.

7. Conclusion

We obtained the action of the D=11 supermembrane compactified on $T^6 \times S^1$ with nontrivial central charge induced by a topological condition invariant under supersymmetric and kappa symmetry transformations. The hamiltonian in the LCG is invariant under conformal transformations on the Riemann surface base manifold. The susy is spontaneously broken, by the vacuum to 1/8 of the original one. It corresponds in 4D to a $N = 1$ multiplet. Classicaly the hamiltonian does not contain singular configurations and at the quantum level the regularized hamiltonian has a discrete spectrum, with finite multiplicity. Its resolvent is compact. The potential does not contain any flat direction on configuration space nor on the moduli space of parameters. The hamiltonian is stable on both spaces. It is stable as a Schrodinger operator on configuration space and it is structurable stable on the moduli space of parameters.

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References

[1] B. de Wit, J. Hoppe and H. Nicolai, *On the quantum mechanics of supermembranes*, Nucl. Phys. **B305** (1988) 545.

[2] B. de Wit, M. Luscher and H. Nicolai, *The supermembrane is unstable*, Nucl. Phys. **B320** (1989) 135.

[3] B. de Wit, U. Marquard and H. Nicolai, *Area preserving diffeomorphisms and supermembrane lorentz invariance*, Commun. Math. Phys. **128** (1990) 39-62.

[4] B. de Wit, K. Peeters and J. Plefka, *Supermembranes with winding.* Phys. Lett.**B409** (1997) 117-123, [arXiv: hep-th/9705225].

[5] M.P. Garcia del Moral and A. Restuccia, *On the spectrum of a noncommutative formulation of the D=11 supermembrane with winding*, Phys.Rev. **D66** (2002) 045023,[arXiv: hep-th/0103261].

[6] L. Boulton, M. P. Garcia del Moral, I. Martin and A. Restuccia, *On the spectrum of a matrix model for the D=11 supermembrane compactified on a torus with non-trivial winding*, Class. Quant. Grav. **19** (2002) 2951,[arXiv:hep-th/0109153].

[7] I. Martin, A. Restuccia and R. S. Torrealba, *On the stability of compactified $D = 11$ supermembranes*, Nucl. Phys. **B521** (1998) 117-128,[arXiv:hep-th/9706090].

[8] L. Boulton, M.P. Garcia del Moral and A. Restuccia, *Discreteness of the spectrum of the compactified D=11 supermembrane with non-trivial winding*, Nucl.Phys. **B671** (2003) 343-358,[arXiv:hep-th/0211047].

[9] J. Bellorin and A. Restuccia, *D=11 Supermembrane wrapped on calibrated submanifolds*, Nucl.Phys. **B737** (2006) 190-208,[arXiv:hep-th/0510259].

[10] L. Boulton, M.P. Garcia del Moral and A. Restuccia, *The Supermembrane with central charges: (2+1)-D NCSYM, confinement and phase transition*, Nucl.Phys. **B795** (2008) 27-51,2008 [arXiv:hep-th/0609054].

[11] J. Gutowski and G. Papadopoulos, *Moduli spaces and brane solitons for M theory compactifications on holonomy G(2) manifolds*, Nucl.Phys. **B615** (2001) 237-265, [arXiv:hep-th/0104105].

[12] A. Lukas and S. Morris, *Moduli Kahler potential for M theory on a G(2) manifold*, Phys.Rev. **D69** (2004) 066003,[arXiv:hep-th/0305078].

[13] B.S. Acharya, *A Moduli fixing mechanism in M theory*, [arXiv:hep-th/0212294].

[14] G. Dall'Agata and N. Prezas, Scherk-Schwarz reduction of M-theory on $G2$-manifolds with fluxes, JHEP **0510** (2005) 103,[arXiv:hep-th/0509052].

[15] B. Acharya, K. Bobkov, G. Kane, P. Kumar and D. Vaman, *An M theory Solution to the Hierarchy Problem*, Phys.Rev.Lett. **97** (2006) 191601,[arXiv: hep-th/0606262]; B. Acharya, K. Bobkov, G. Kane, P. Kumar and J. Shao,*Explaining the Electroweak Scale and Stabilizing Moduli in M Theory*, [arXiv:hep-th/0701034].

[16] I. Martin, J. Ovalle and A. Restuccia, *D-branes, symplectomorphisms and noncommutative gauge theories.* Nucl. Phys. Proc. Suppl. **102** (2001) 169-175, *Compactified D = 11 supermembranes and symplectic noncommutative gauge theories* , Phys. Rev. **D64** (2001) 046001, [arXiv:hep-th/0101236].

[17] A. Belhaj, M.P. Garcia del Moral, I. Martin, A. Segui, J. Veiro and A. Restuccia, *The Supermembrane with Central Charges on a G2 Manifold*, [arXiv:hep-th/0803.1827].

[18] E. Bergshoeff, E. Sezgin and P.K. Townsend, *Supermembranes and eleven-dimensional supergravity*, Phys. Lett. **B189** (1987) 75-78.

[19] E. Witten *On S duality in Abelian gauge theory*, Selecta Math.**1** (1995) 383,[arXiv:hep-th/9505186].

[20] M.I. Caicedo, I. Martin and A. Restuccia, *Gerbes and duality.* Annals Phys. **300** (2002) 32-53, [arXiv:hep-th/0205002].

[21] J. Bellorin, A. Restuccia *Extended selfdual configurations as stable exact solutions in Born-Infeld theory*, Phys.Rev. **D64** (2001) 106003,[arXiv:hep-th/0007066].

[22] D. J. Smith *Intersecting brane solutions in string and M theory.* Class.Quant.Grav. **20** (2003) R233,[arXiv:hep-th/0210157].

[23] L. Boulton and A. Restuccia, *The Heat kernel of the compactified D=11 supermembrane with non-trivial winding*, Nucl. Phys. **B724** (2005) 380-396, [arXiv:hep-th/0405216].

[24] M.J. Duff, T. Inami, C.N. Pope, E. Sezgin and K.S. Stelle, *Semiclassical Quantization Of The Supermembrane*, Nucl. Phys. **B297** (1988) 515,

[25] J. Scherk and J.H. Schwarz *How to get masses from extra dimensions*, Nucl.Phys.**B153** (1979) 61-68.

[26] Y. Hosotani *Dynamical mass generation by compact extra dimensions*, Phys.Lett**B126** (1983) 309.

[27] M.Sakamoto, M. Tachibana, K. Takegana *Spontaneous supersymmetry breaking from extra dimensions*, Phys.Lett. **B458** (1999) 231-236,[arXiv:hep-th/9902070].

[28] M.E. Shaposhnikov and P. Tinyakov *Extra dimensions as an alternative to Higgs mechanism*, Phys.Lett. **B515** (2001) 442-446,[arXiv:hep-th/0102161].

[29] T. Friedmann *On the quantum moduli space of M theory compactifications*, Nucl.Phys.**B635** (2002) 384-394,[arXiv:hep-th/0203256].

[30] M.P. Garcia del Moral, A. Restuccia, *The Supermembrane with central charge as a bundle of D2 - D0 branes*, Institute of Physics Conference Series 2005, Vol **43**, 151, [arXiv:hep-th/0410288].

[31] J. Evslin and L. Martucci *D-brane networks in flux vacua, generalized cycles and calibrations*, JHEP **0707** (2007) 040,[arXiv:hep-th/0703129].